Cosmology and the origin of structure

Rocky I: The universe observed

Rocky II: The growth of cosmological structure

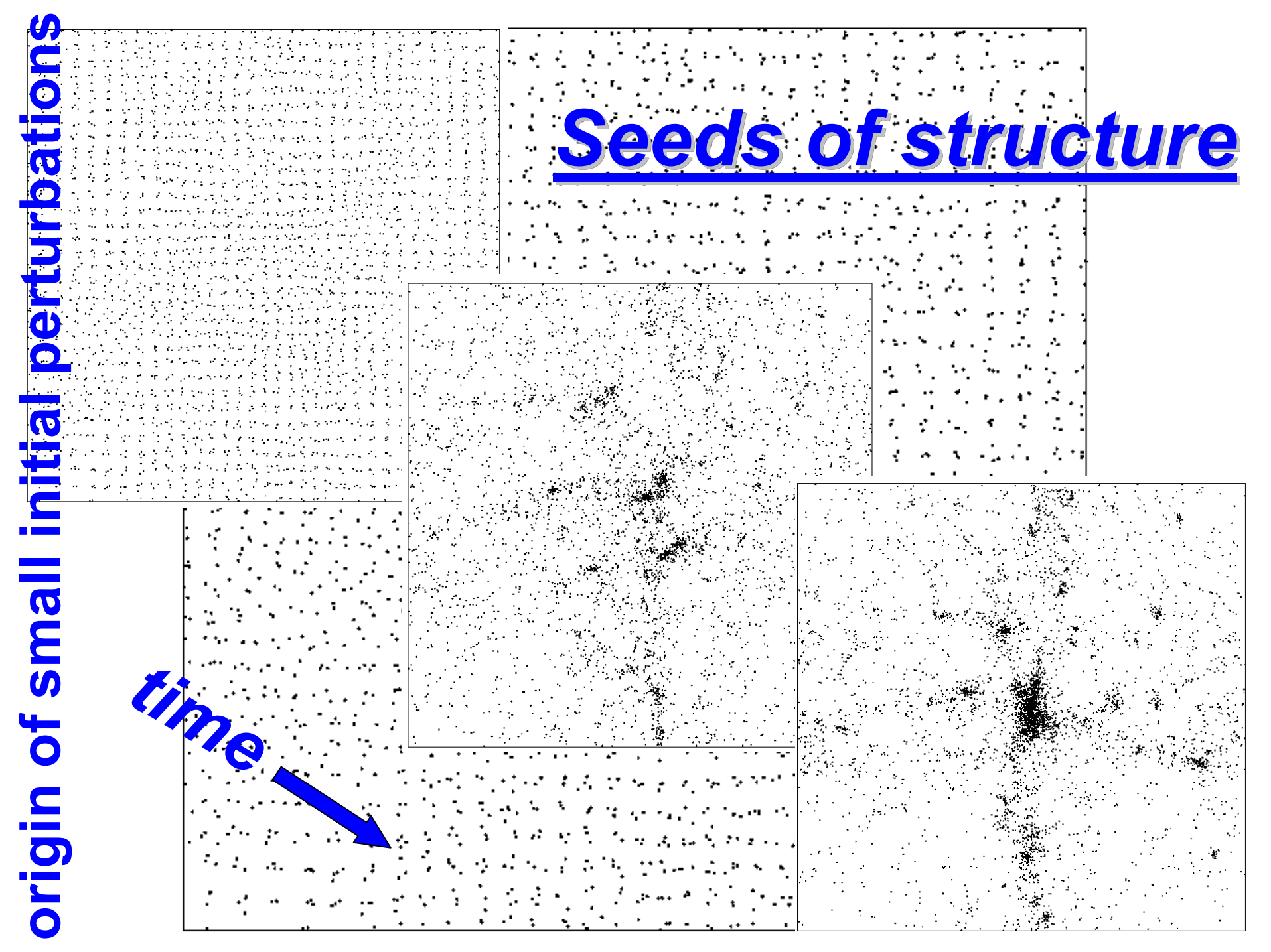
Rocky III: Inflation and the origin of perturbations

Rocky IV: Dark matter and dark energy

Academic Training Lectures

Rocky Kolb

Fermilab, University of Chicago, & CERN

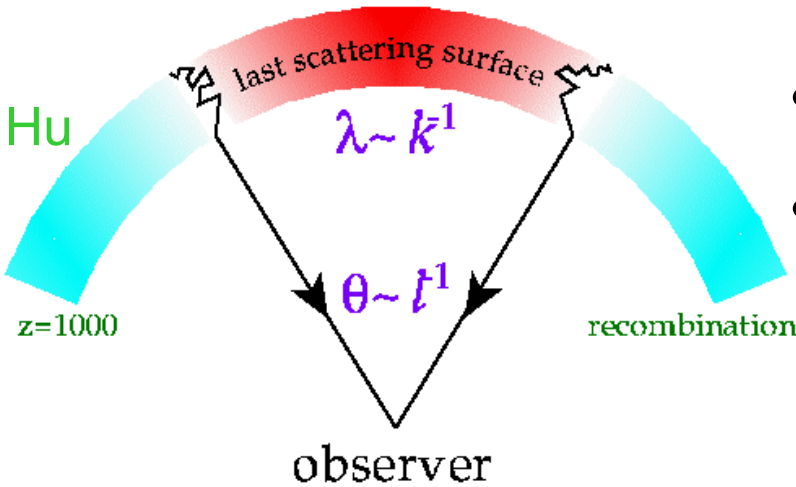

origin of small initial perturbations
Seeds of structure
time

Rocky III: Inflation

- Motivation for “acausal” perturbations
- Inflation in the cosmic symphony
- The alarming phenomenon of particle creation
- Inflation phenomenology

Acoustic peaks

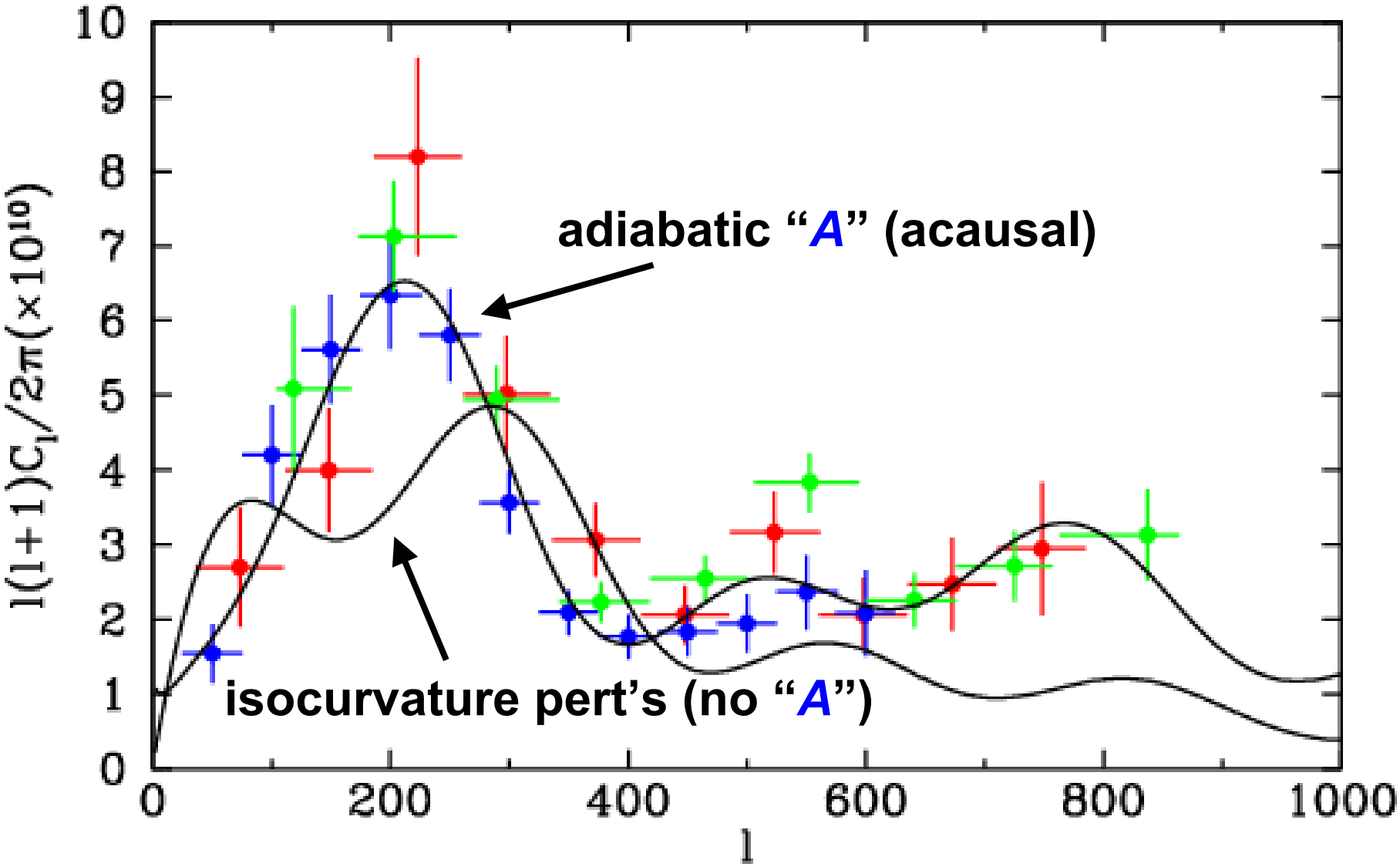
- At recombination, baryon–photon fluid undergoes "acoustic oscillations" $A\cos kt + B\sin kt$
- Compressions and rarefactions change T_γ
- Peaks in ΔT_γ correspond to extrema of compressions and rarefactions



- What are coefficients A & B ?
- A nonzero ⟹ perturbations in limit $kt_{\text{last-scattering}} \ll 1$

$$kt_{\text{last-scattering}} \ll 1 \Rightarrow \frac{t_{\text{last-scattering}}}{\lambda} \ll 1 \Rightarrow t_{\text{last-scattering}} \ll \lambda$$

Angular power spectrum



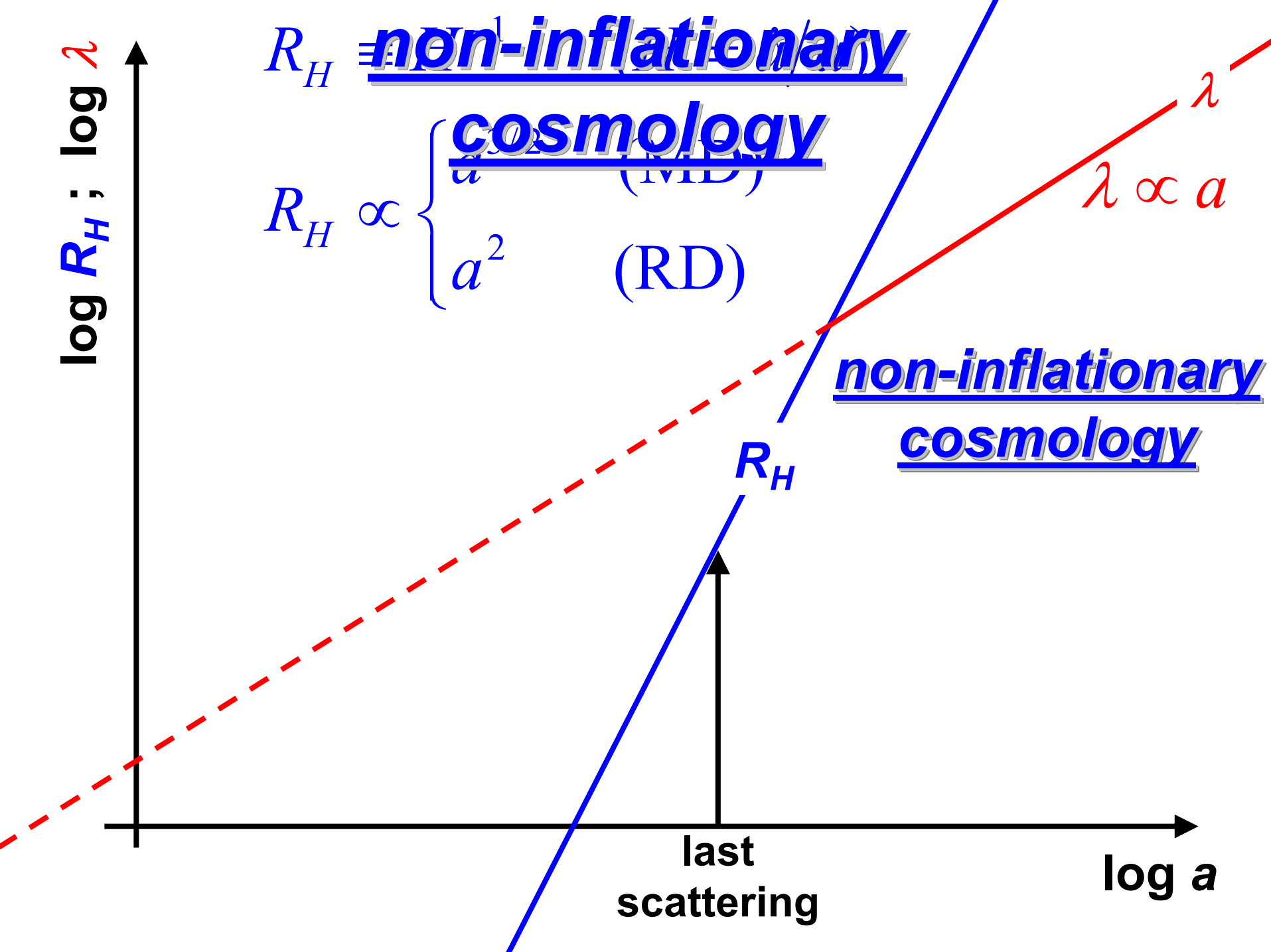
Input from the early universe

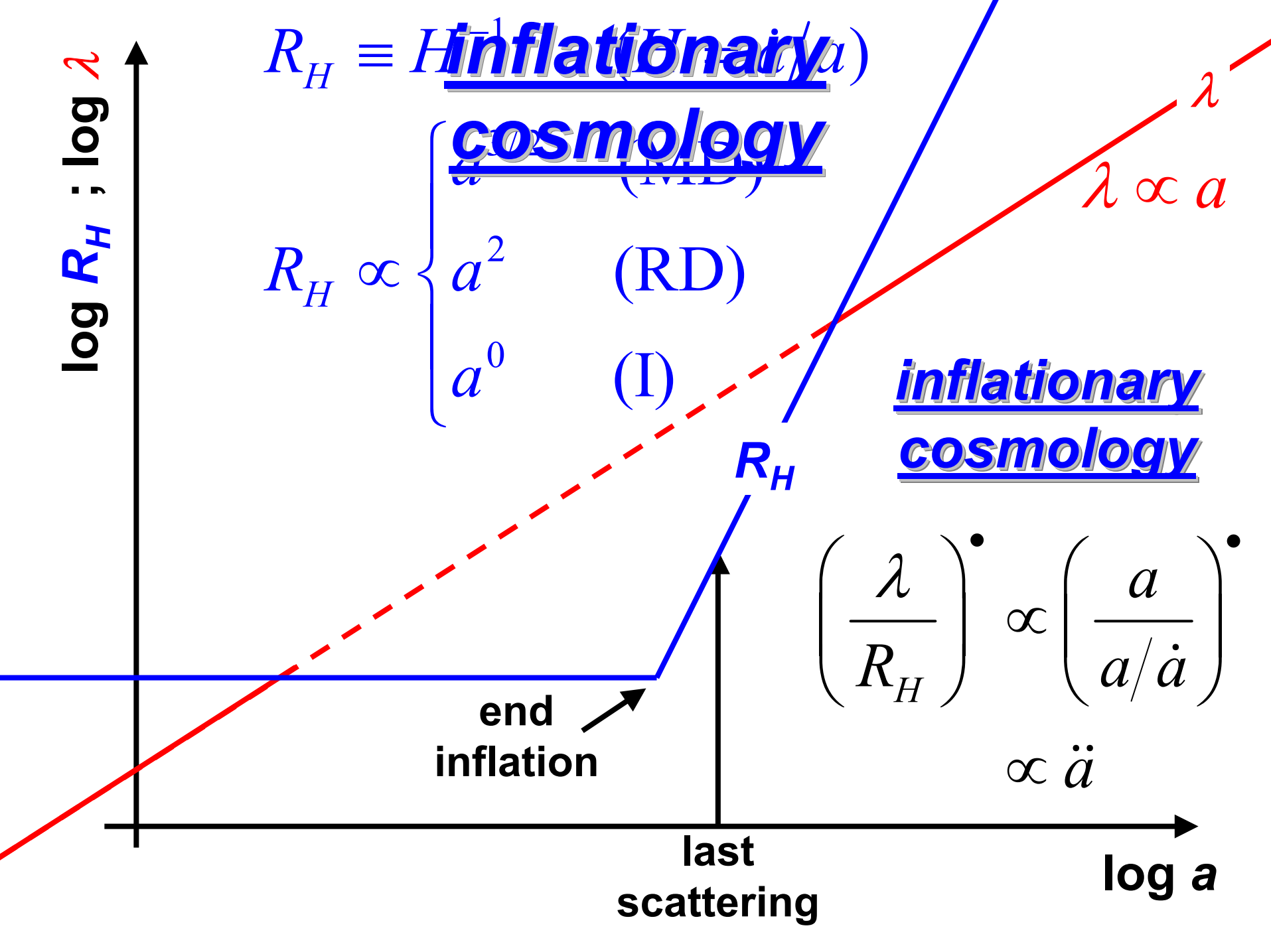
1. Spectrum of perturbations (power spectrum)

   CMB correlations on scales greater than Hubble radius ($R_H \sim ct$) ⟶ INFLATION

2. Matter content

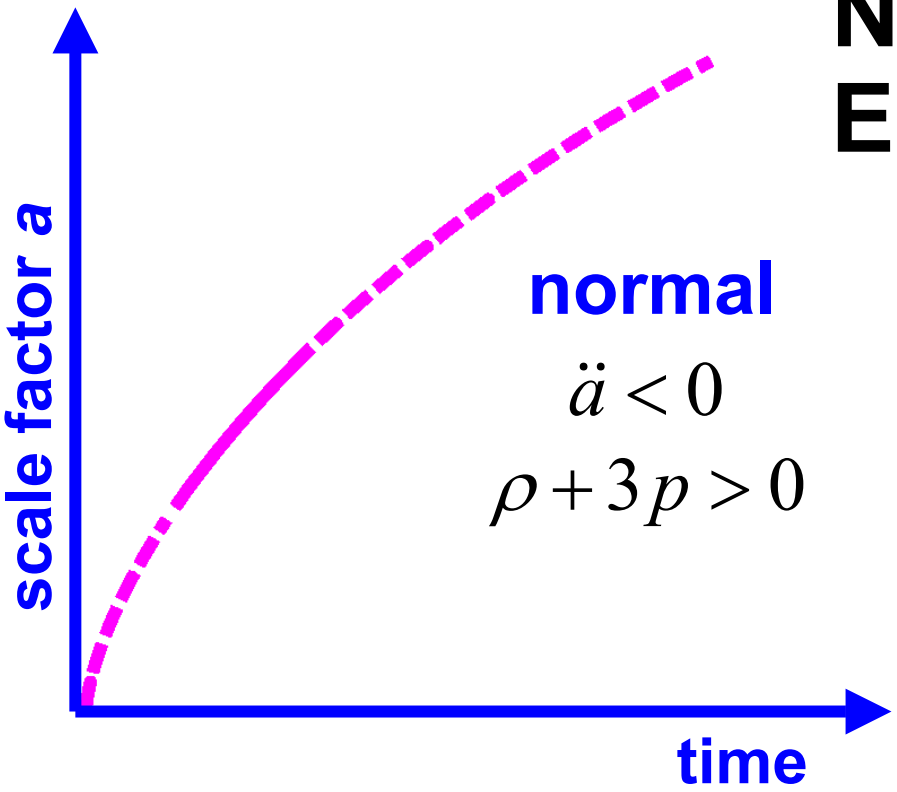
   baryons, dark matter (cold, warm, hot), vacuum energy, ……


non-inflationary cosmology
$R_H \equiv$
$R_H \propto$
a^2 (RD)
λ
$\lambda \propto a$
non-inflationary cosmology
R_H
log R_H ; log λ
last scattering
log a


$R_H \equiv H^{-1} \propto t/a$
$R_H \propto \begin{cases} a^2 & \text{(RD)} \\ a^0 & \text{(I)} \end{cases}$
inflationary cosmology
inflationary cosmology
$\log R_H$; $\log \lambda$
λ
$\lambda \propto a$
R_H
$\left(\frac{\lambda}{R_H}\right)^{\bullet} \propto \left(\frac{a}{a/\dot{a}}\right)^{\bullet}$
$\propto \ddot{a}$
end inflation
last scattering
$\log a$

Newton
Einstein

$$\ddot{a} \propto -G\left(\rho + 3p\right)$$

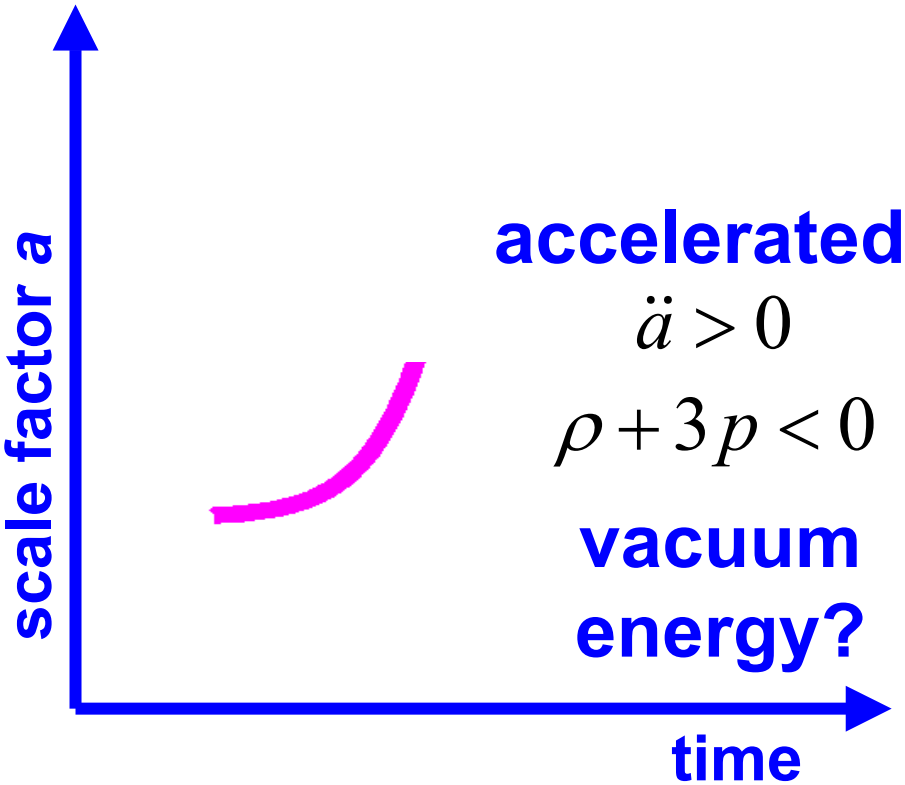


normal

$$\ddot{a} < 0$$

$$\rho + 3p > 0$$

ρc^2 and $p c^2$



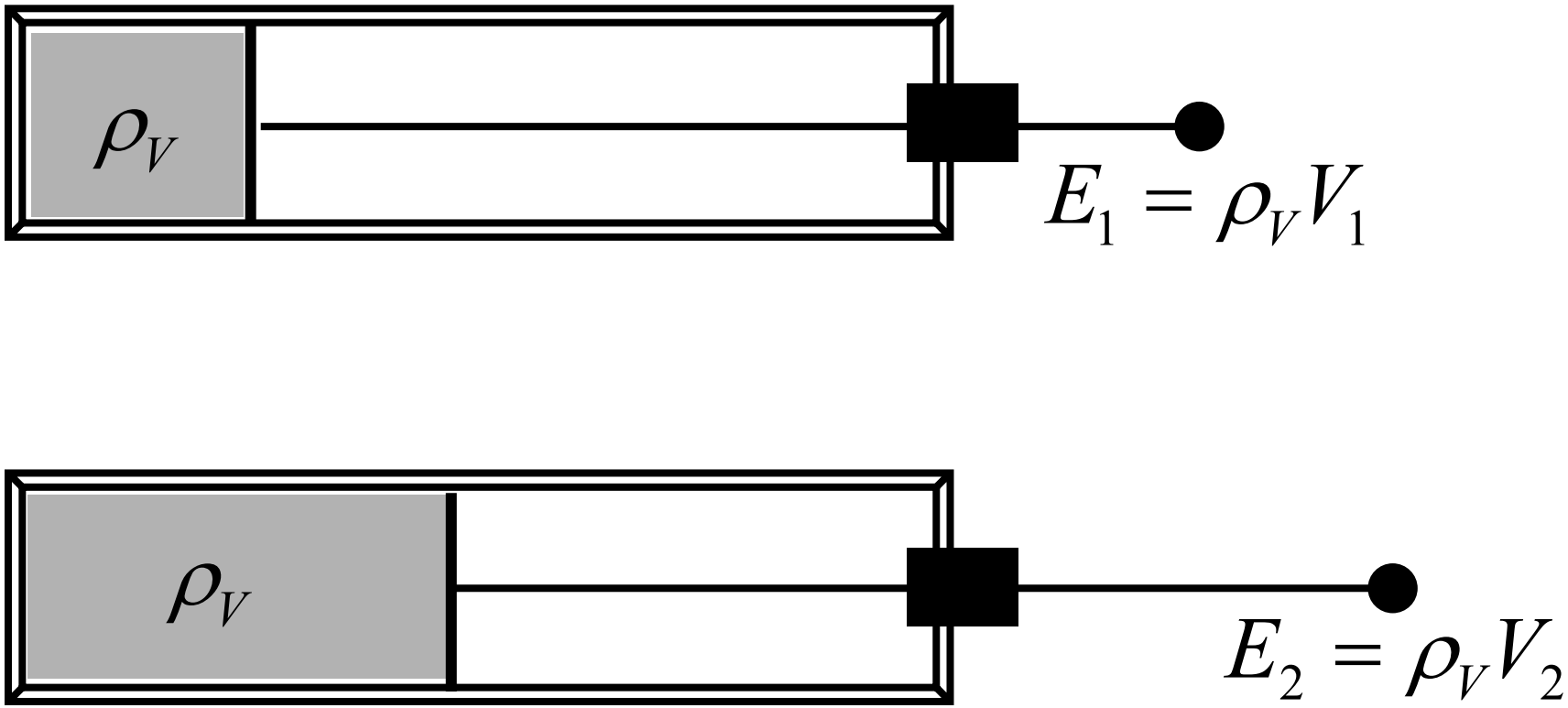
accelerated

$$\ddot{a} > 0$$

$$\rho + 3p < 0$$

vacuum energy?

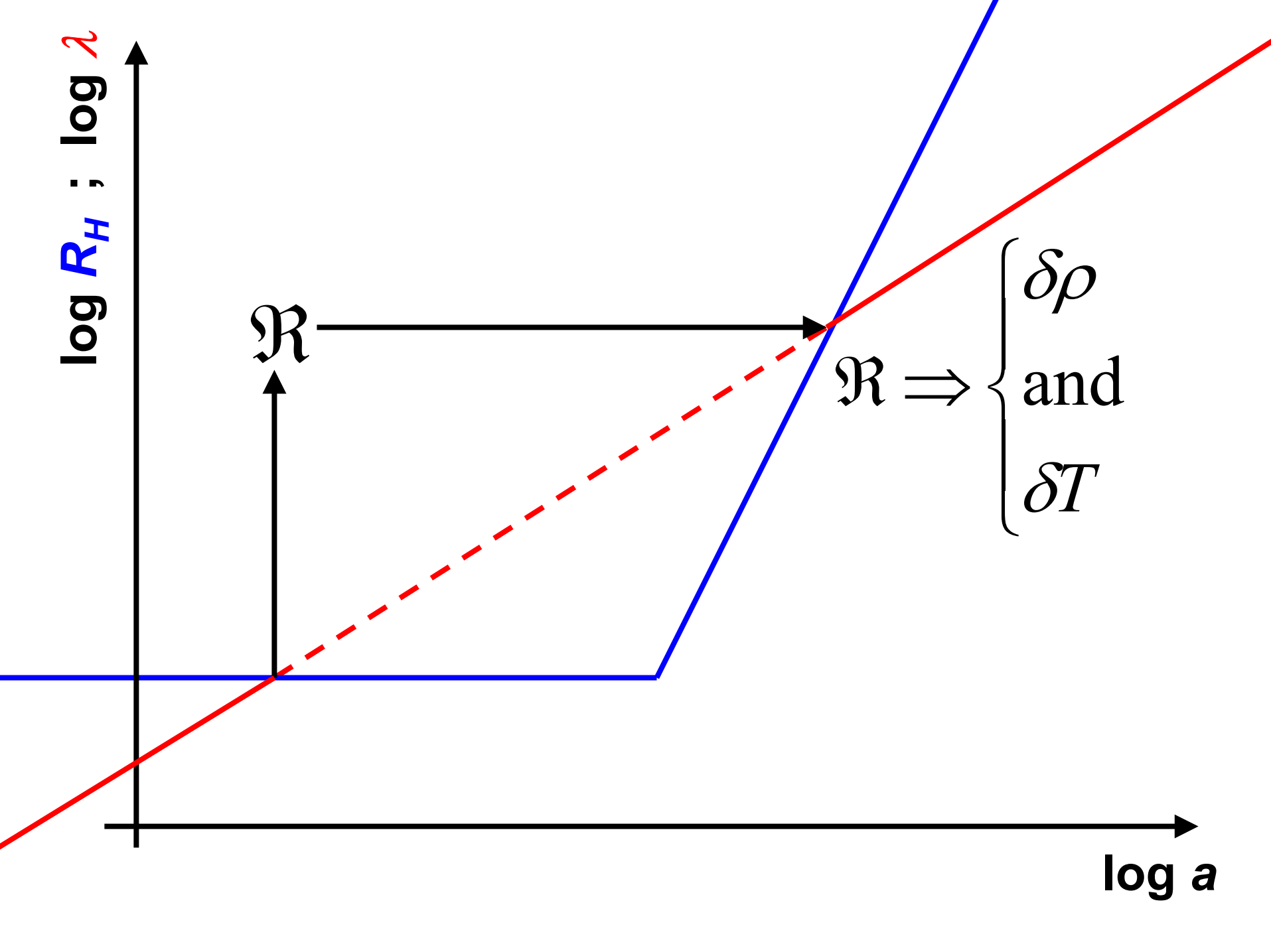
Vacuum pressure



$E_2 > E_1$

had to *pull* piston

“negative pressure”


log R_H ; log λ
$\Re$
$\Re \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
log a

Cosmic Symphony (Harmonice Mundi)

expansion tempo	movement	epoch	relic
pizzicato	string dominated	10^{-43} sec.?	???
presto	inflation $H \propto a^0$	10^{-35} sec.?	CBR fluctuations, gravitational waves, seeds of structure
allegro	radiation dominated $H \propto a^{-2}$	earlier than 10,000 yrs.	abundance of the light elements
andante	matter dominated $H \propto a^{-3/2}$	later than 10,000 yrs.	distant quasars and galaxies
largo	inflation $H \propto a^0$	day before yesterday	acceleration of the universe

Inflation, as a whole, can be divided into three parts

1. **Beginning**

   eternal inflation, wave function of the universe, did the universe have a beginning ????

2. **Middle**

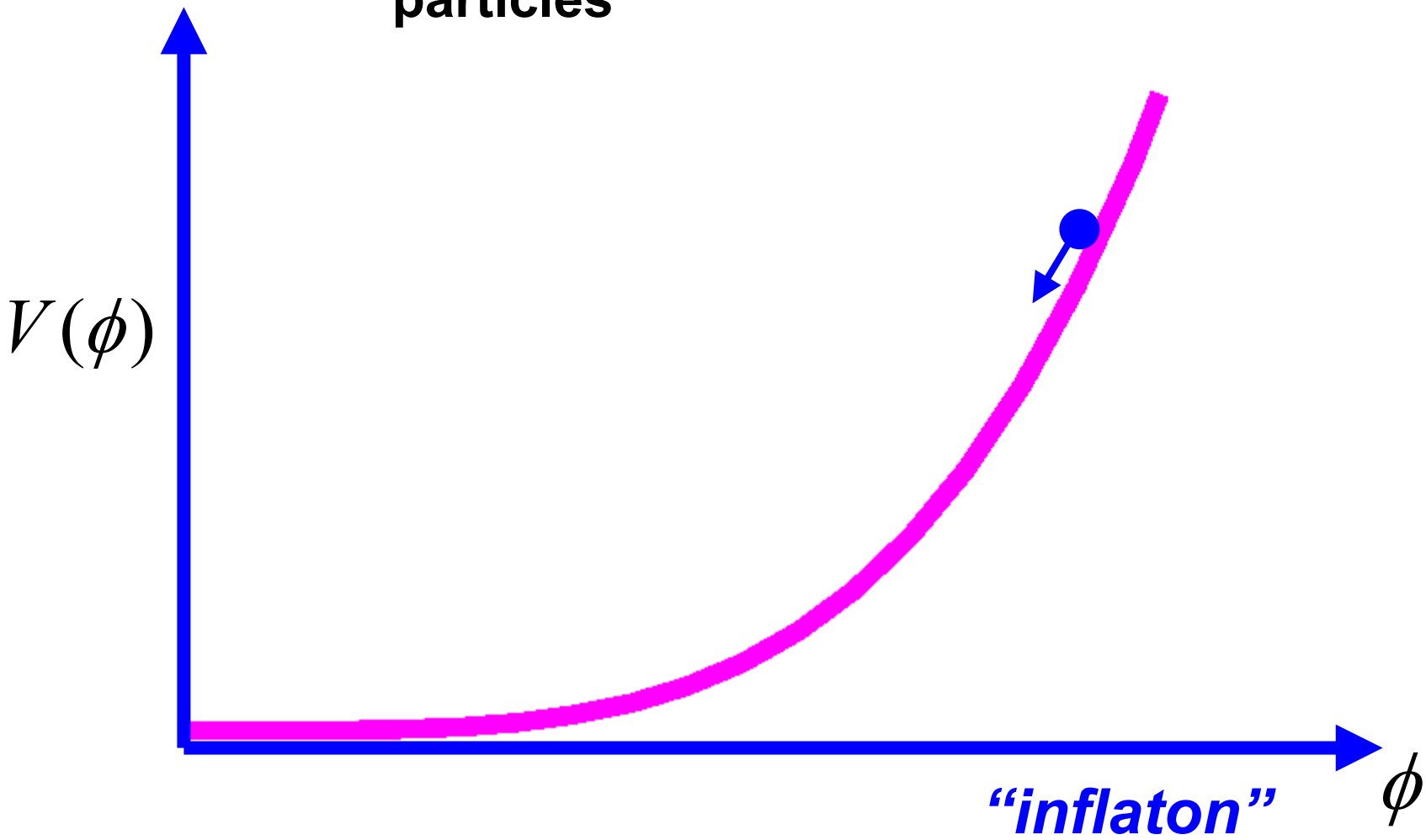
   density perturbations, gravitational waves, (particle production in the expanding universe)

3. **End**

   defrosting, heating, preheating, reheating, baryogenesis, phase transitions, dark matter, (particle production in the expanding universe)

Potential energy: energy of infinite-wavelength mode

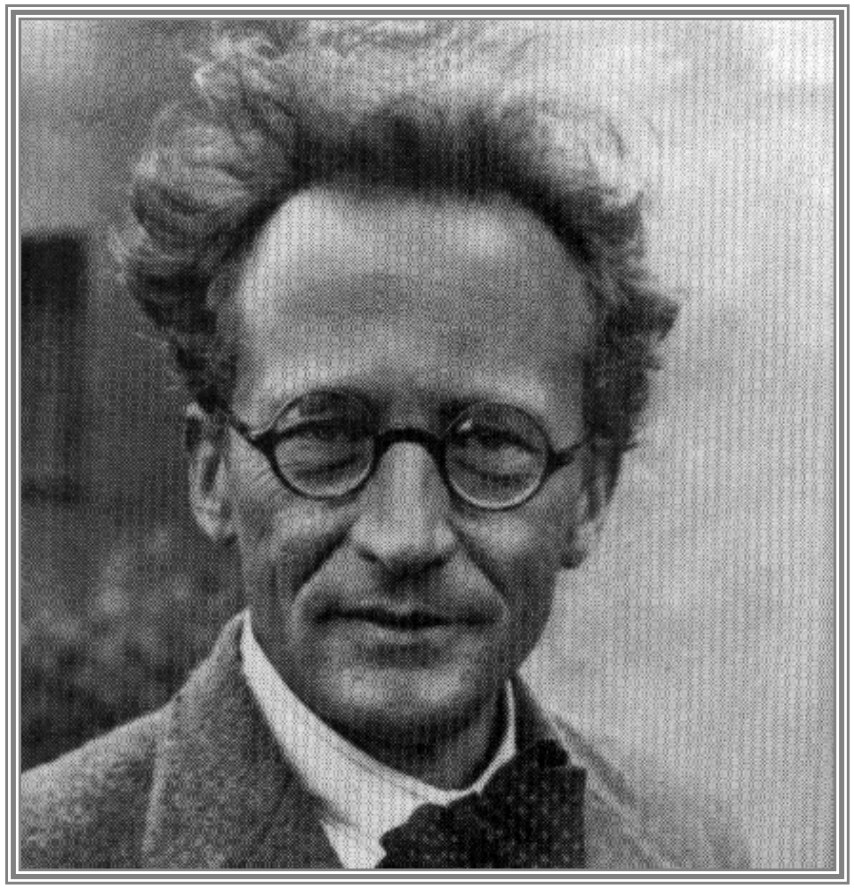
Particle content: condensate of infinite-wavelength particles



Classical equations of motion

$$V(\phi) \neq 0 \longrightarrow V(\phi) = 0$$

An early particle cosmologist



In mid-1930s, Schrödinger turned to cosmo issues, influenced by Eddington & Lemaitre

1938-1939: Graz → Vatican → Gent, Belgium → Dublin

The proper vibrations of the expanding universe

Erwin Schrodinger

Physica **6**, 899 (1939)

Introduction:

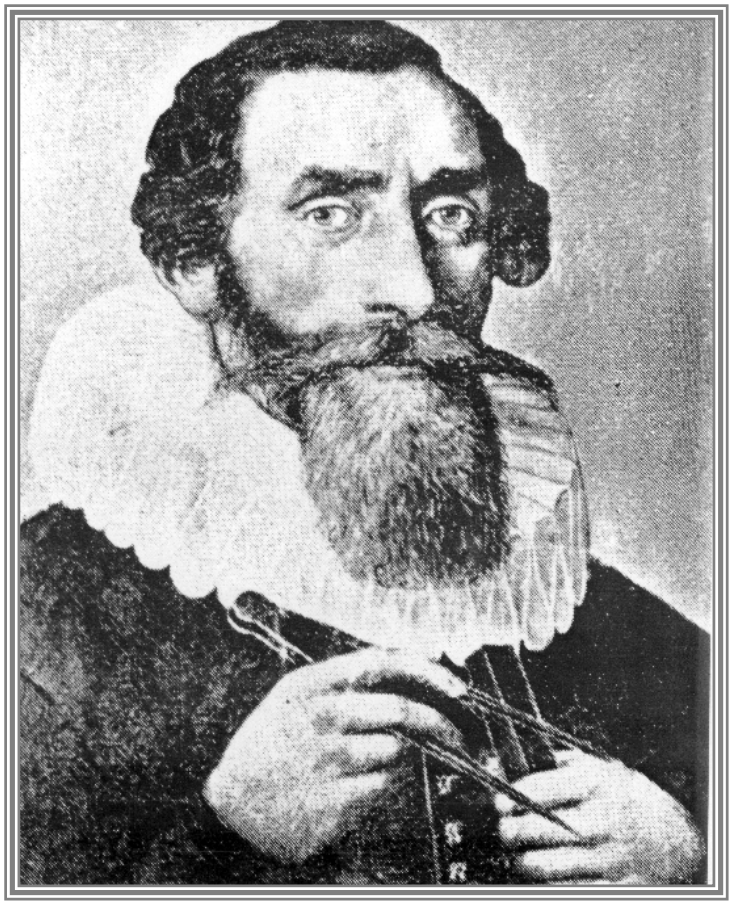
"… proper vibrations [positive and negative frequency modes] cannot be rigorously separated in the expanding universe.

… this is a phenomenon of outstanding importance. With particles it would mean production or annihilation of matter, merely by expansion,… Alarmed by these prospects, I have examined the matter in more detail."

Conclusion:

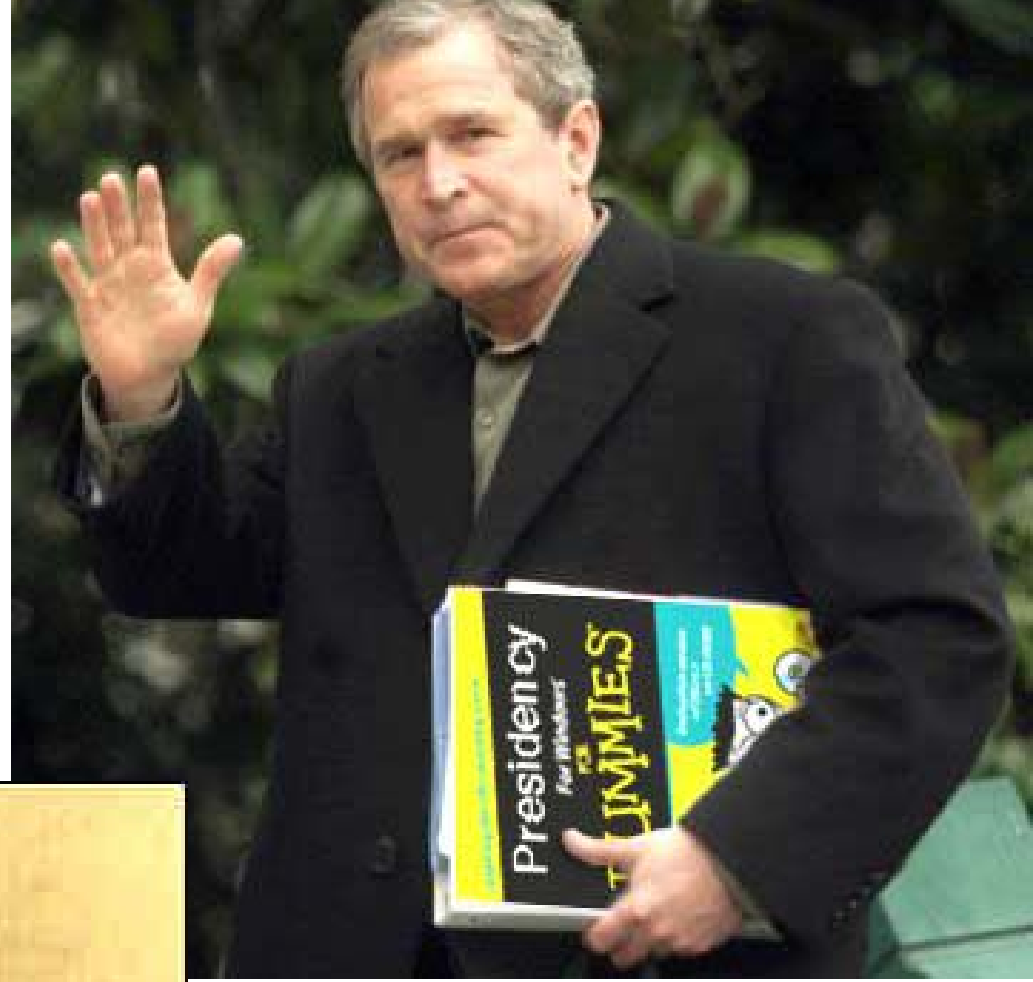
"… There will be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to … the 'alarming phenomenon'…"

An even earlier Graz cosmologist



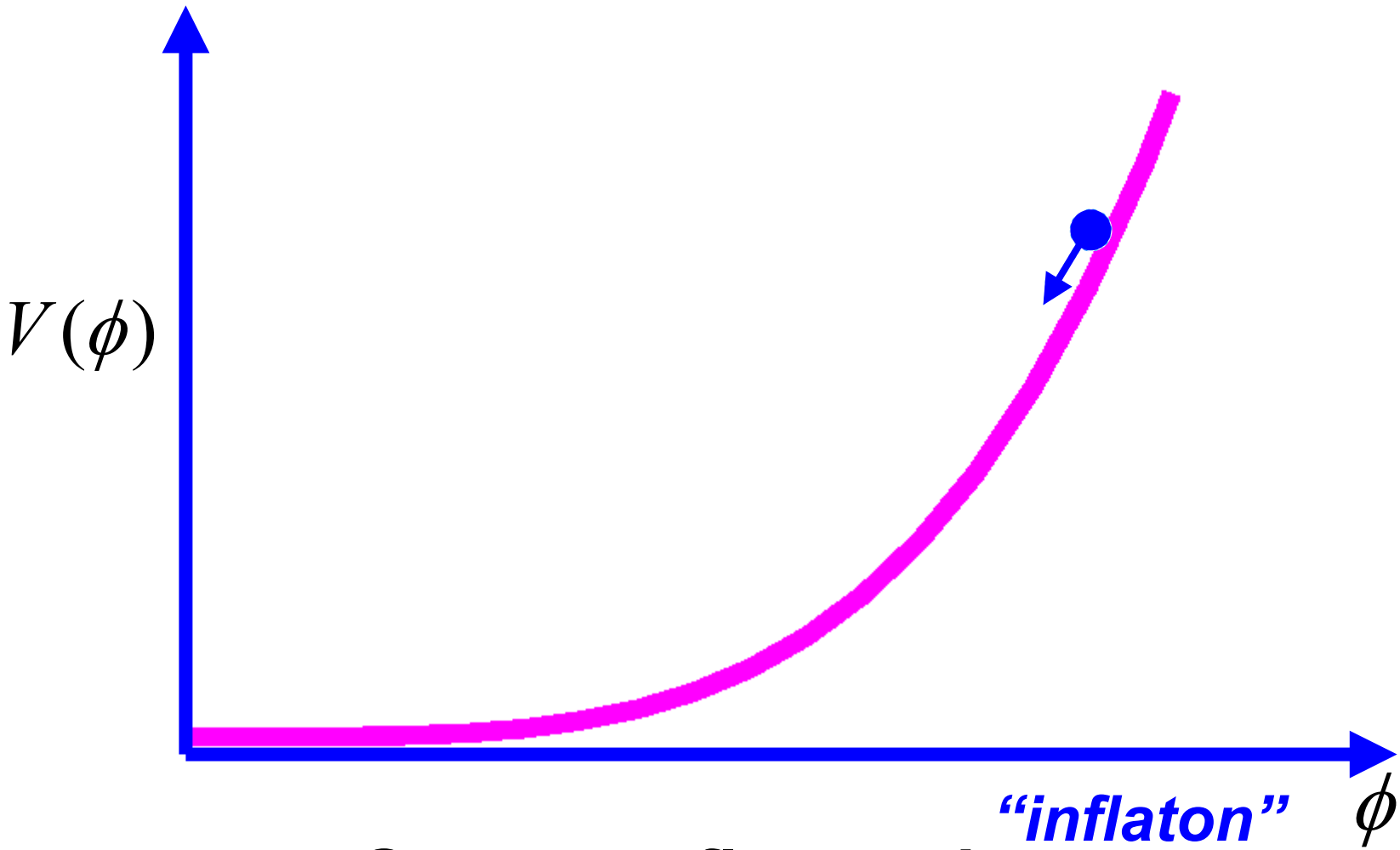
"When the storms rage around us, and the state is threatened by shipwreck, we can do nothing more noble than to lower the anchor of our peaceful studies in the ground of eternity." - *J. Kepler*

1600-1630: Graz → Prague → Linz → Sagan → Ratisbon


Presidency
for Dummies

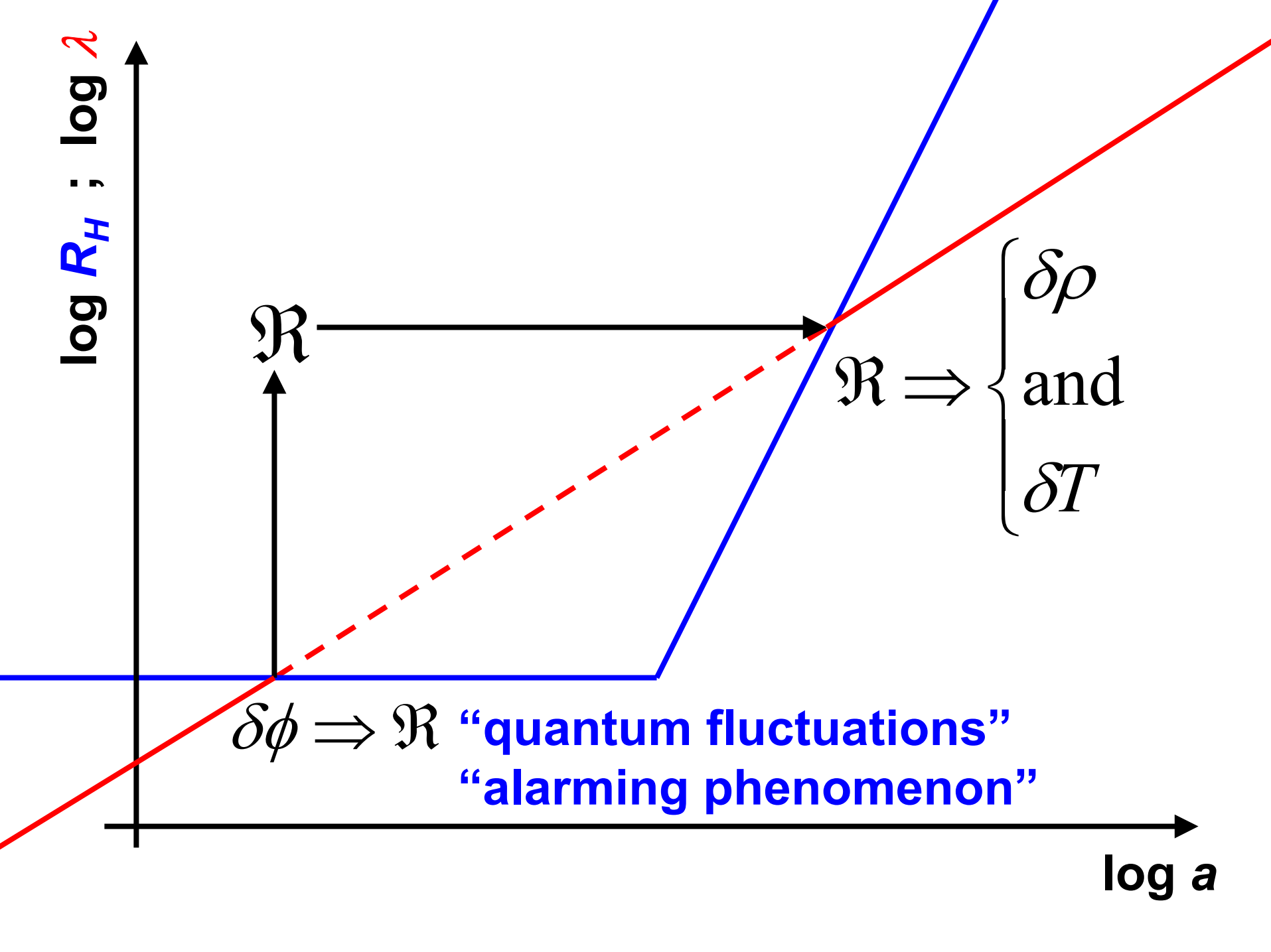
Particle creation: finite-wavelength modes

$\longrightarrow \phi$ **not smooth**

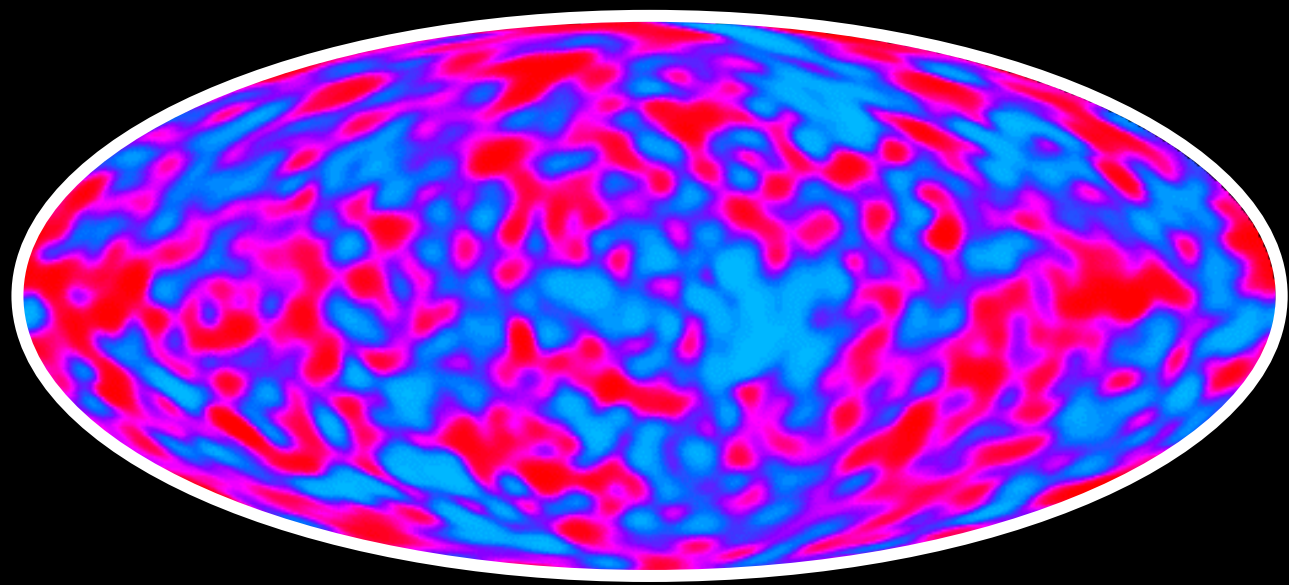


Quantum fluctuations

$\delta\phi \longrightarrow \delta\rho \longrightarrow \delta T$


log R_H ; log λ
$\Re$
$\Re \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
$\delta\phi \Rightarrow \Re$ "quantum fluctuations"
"alarming phenomenon"
log a

patterns of quantum fluctuations



Variational Formalism for Quantization:

$$S = \int d^4x\sqrt{-g}\left[-\frac{M_{Pl}^2}{16\pi}R + \tfrac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$g_{\mu\nu}(\vec{x},t) = g_{\mu\nu}^{FRW}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$\phi(\vec{x},t) = \phi_0(t) + \delta\phi(\vec{x},t)$$

Scalar perturbations in terms of a field *u*

$$u \propto \delta\phi + \delta g_{\mu\nu}^{SCALAR}(\vec{x},t)$$

$$S = \int d^4x\left(\partial_\mu u\partial^\mu u - \tfrac{1}{2}m^2u^2\right)$$

Minkowski space (conformal time)

$$m^2 = -z^{-1}\,d^2z/d\eta^2$$

mass changes with time

$$z = a\phi_0'/H$$

Variational formalism for quantization:

Einstein gravity → ← Inflaton field

$$S = \int d^4x\sqrt{-g}\left[-\frac{M_{Pl}^2}{16\pi}R + \tfrac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$g_{\mu\nu}(\vec{x},t) = g_{\mu\nu}^{\mathrm{FRW}}(t) + \delta g_{\mu\nu}^{\mathrm{tensor}}(\vec{x},t)$$

Tensor perturbations in terms of $v(h_{ij})$

$v \propto$ gravitons

$$S = \int d^4x\left(\partial_\mu v\partial^\mu v - \tfrac{1}{2}m_v^2 v^2\right)$$

Minkowski space (conformal time)

m_v^2 **changes in time**

Quantum generation of perturbations:

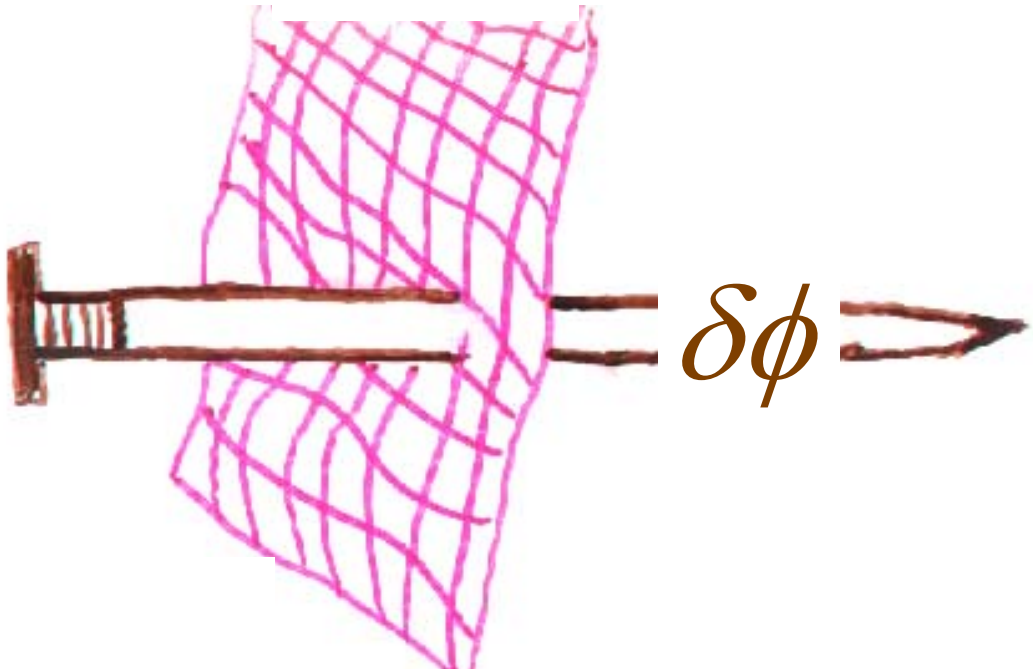
- **Wave equation for *u***

$$\frac{d^2 u_k}{d\eta^2} + \left(k^2 - \frac{1}{z}\frac{d^2 z}{d\eta^2} \right) u_k = 0 \qquad \left(z = a\phi_0' / H \right)$$

- **Initially only homogeneous** $(k = 0)$ **mode.**
- **As evolve, mass is complicated function of time.**
- **Create nonzero momentum mode.**
- ***Alarming phenomenon!***

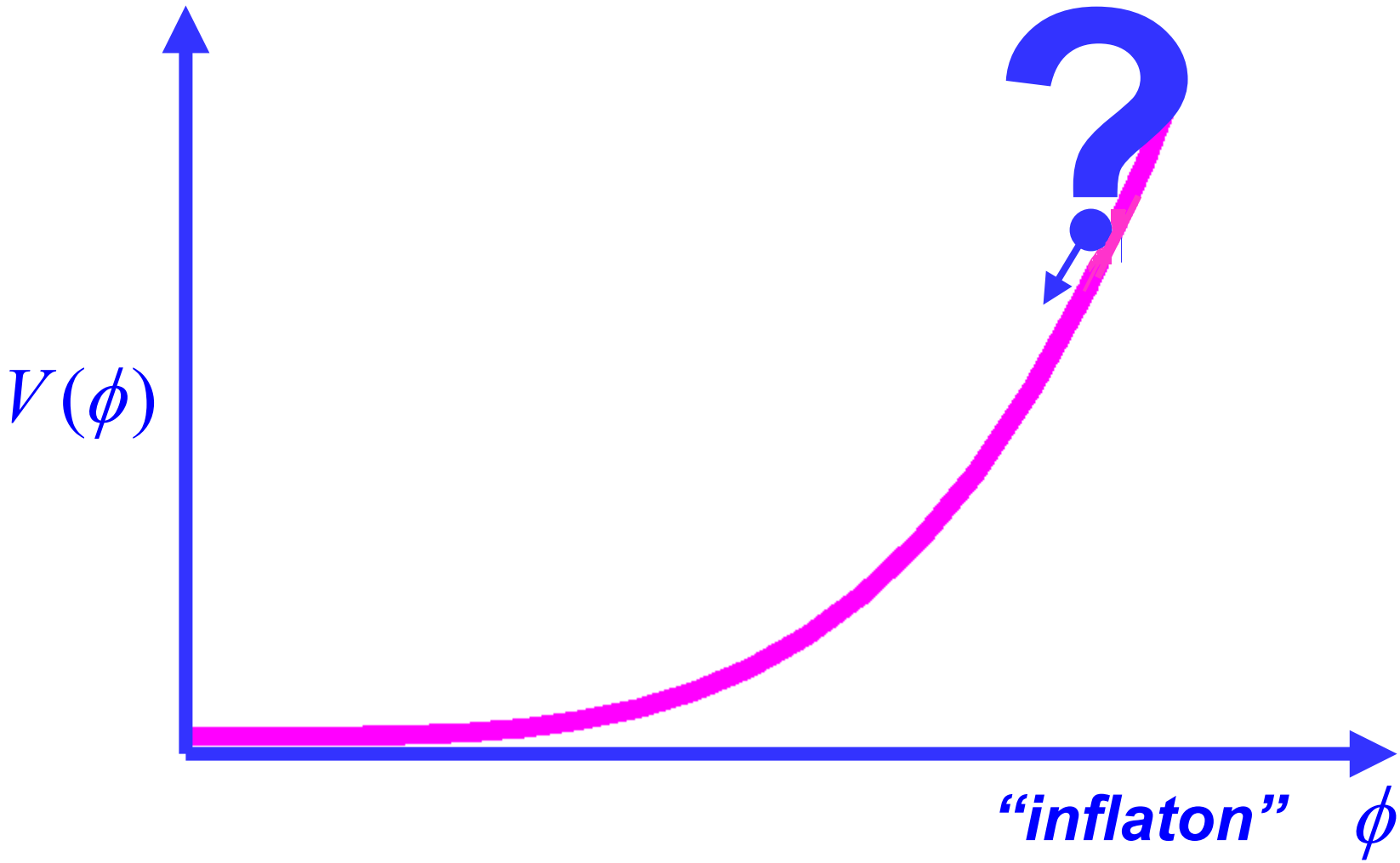
$$u_k \rightarrow \Re_k \rightarrow \text{Power Spectrum}$$

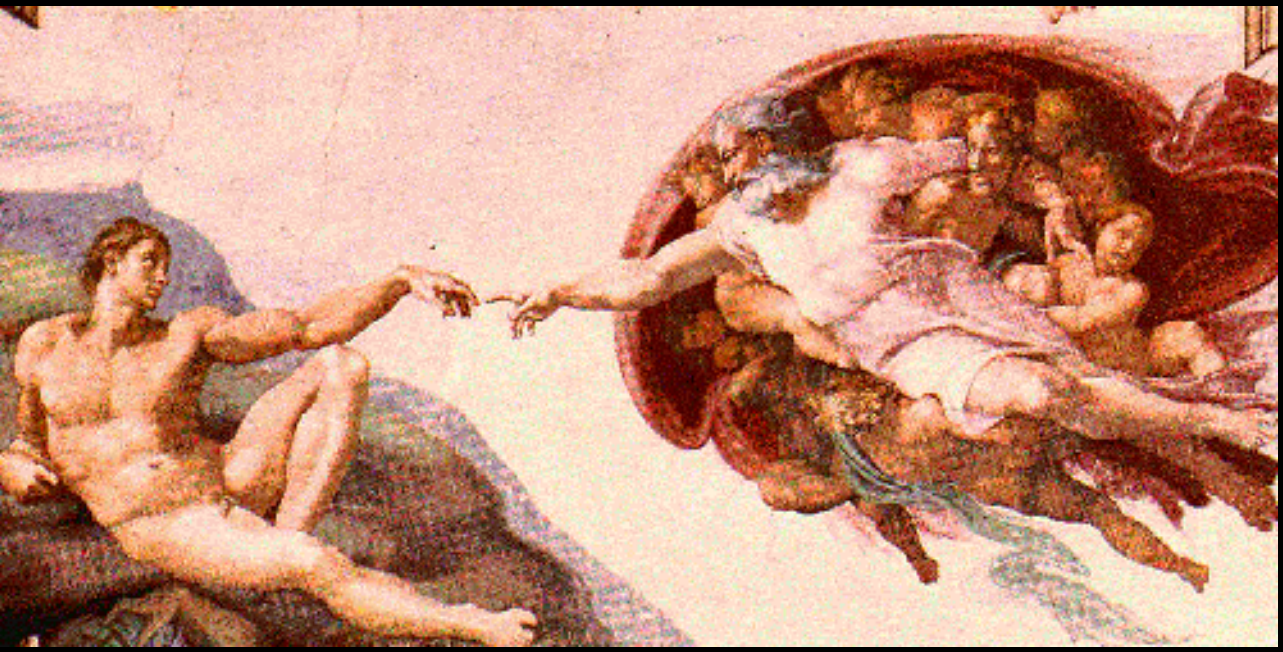
$\delta g_{\mu\nu}$



(When a hammer is your only tool, everything has the appearance of a nail.)

Who is the inflaton?





Top down

Bottom up

Models of inflation

old, new, pre-owned,
chaotic, quixotic, ergodic,
ekpyrotic, autoerotic,
faith-based, free-based,
brane, braneless, brainless,
supersymmetric, supercilious,
natural, supernatural, *au natural,*
hybrid, low-bred, white bread,
one-field, two-field, left-field,
eternal, internal, infernal,
self-reproducing, self-promoting,
dilaton, dilettante, …….

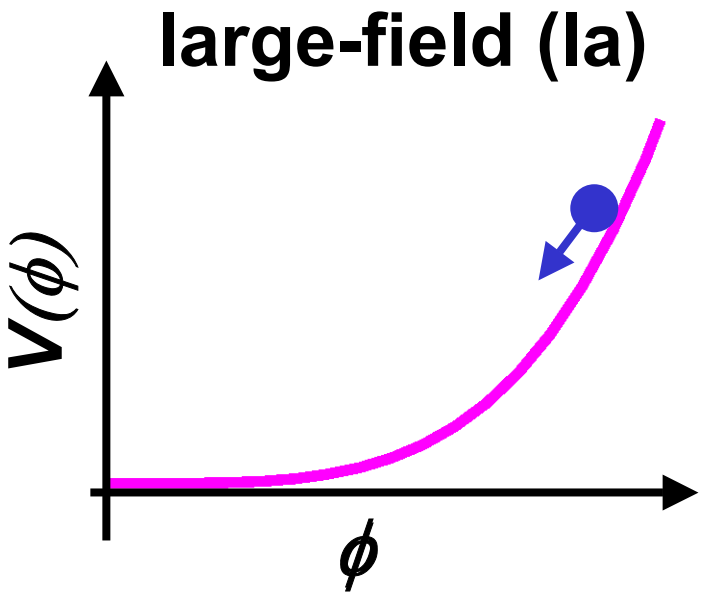
Model Classification*

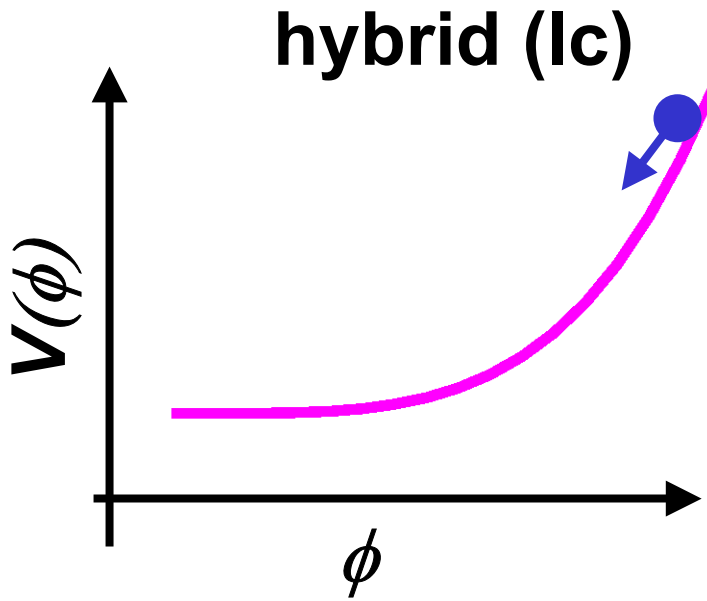
Type I: single-field, slow-roll models
(or models that can be expressed as such)

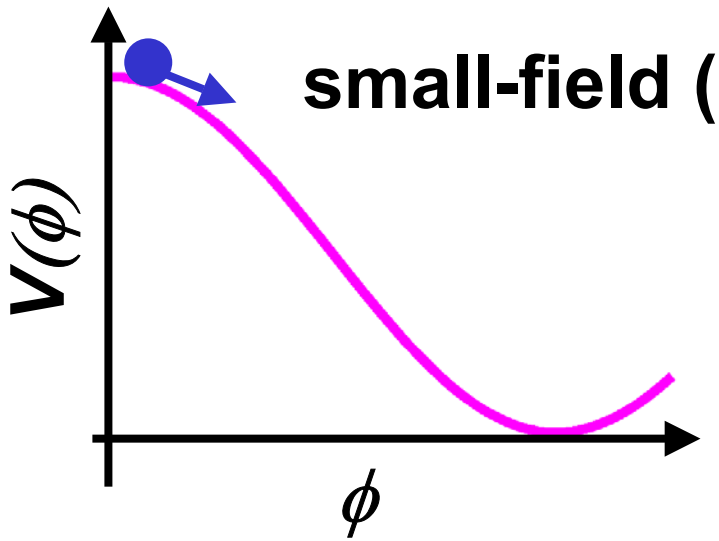
- **Type Ia:** large-field models
- **Type Ib:** small-field models
- **Type Ic:** hybrid models

Type II: anything else
(branes, pre-big-bang, etc.)

**Used for superstrings, supernovae, superconductors, …*


large-field (la)
V(ϕ)
ϕ


hybrid (lc)
V(ϕ)
ϕ


small-field (lb)
V(ϕ)
ϕ

Quantum generation of perturbations:

- **Perturbations model-dependent function of H and how H changes during inflation.**

$$V(\phi) \leftrightarrow H(\phi)$$

- **Characterize perturbations in terms of:**

$A_s(k_*)$ scalar perturbation at $k = k_*$

$n \equiv \dfrac{d \ln A_s^2(k_*)}{d \ln k}$ scalar spectral index

$A_T(k_*)$ tensor perturbation at $k = k_*$

$n_T \equiv \dfrac{d \ln A_T^2(k_*)}{d \ln k}$ tensor spectral index

Quantum generation of perturbations:

- **Input inflation potential** $V(\phi)$ **:**

$$\{A_S(k_*) \quad n \quad A_T(k_*) \quad n_T\}$$

- **Observer-friendly parameters:**

$$Q \equiv A_S^2(k_*) + A_T^2(k_*) \qquad n \qquad r \equiv \frac{A_T^2(k_*)}{A_S^2(k_*)}$$

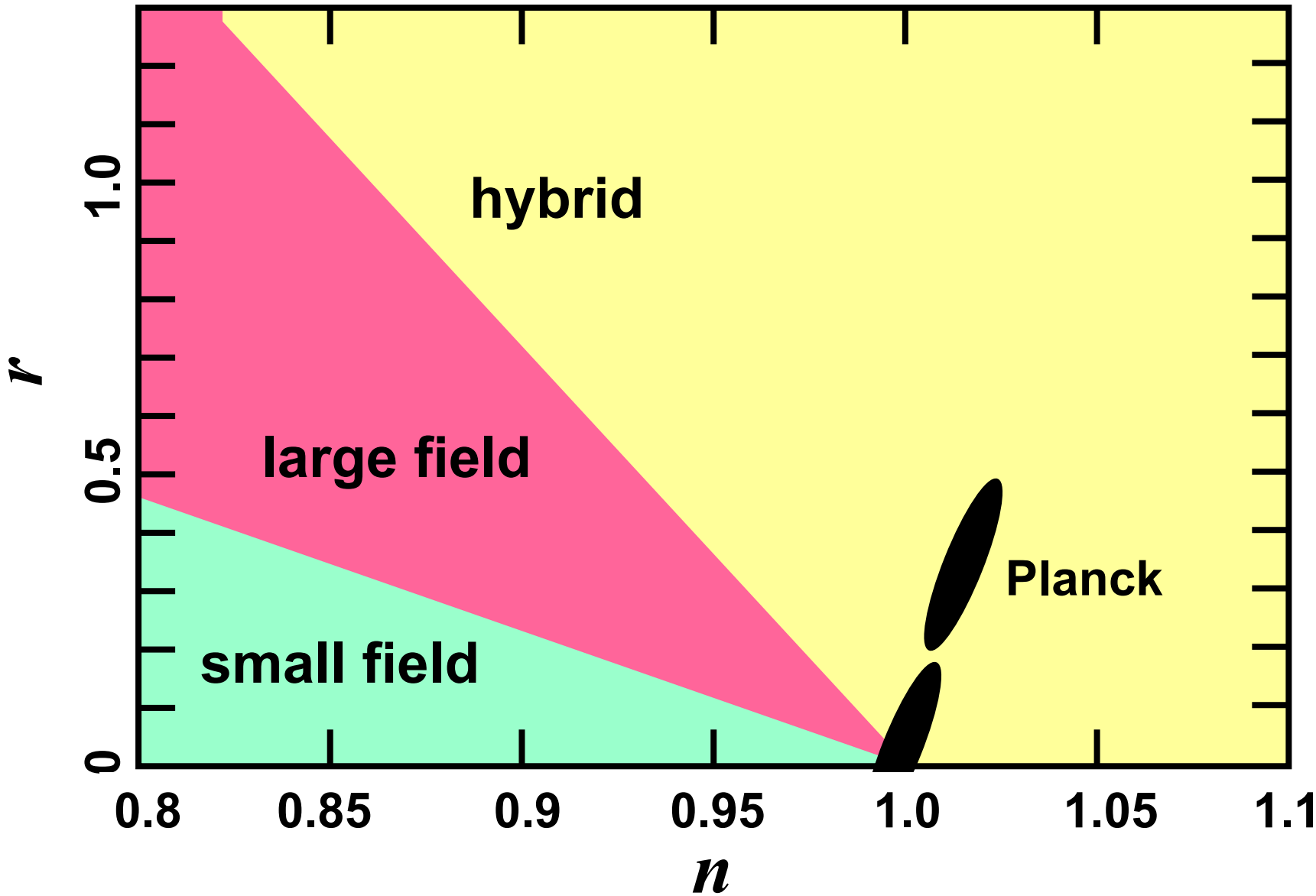
- **Consistency relation:**

$$n_T = -2r\left[1 - r + (1 - n)\right]$$

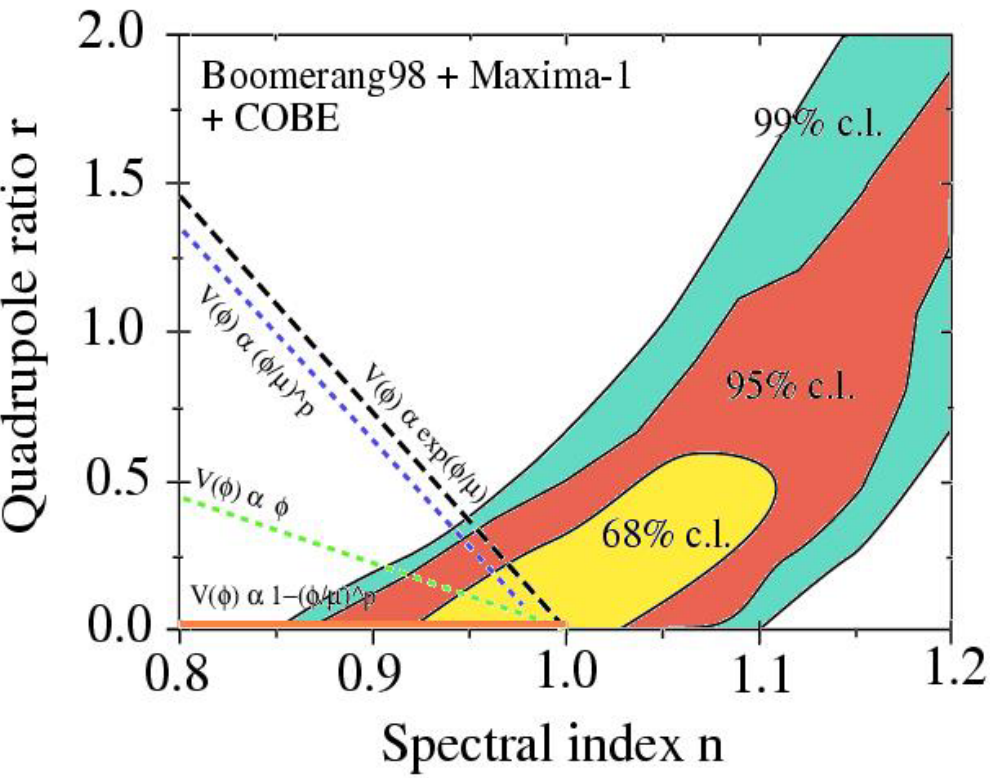
- **Inflaton potential** $V(\phi)$**:**

$$\{Q \quad n \quad r\} \text{ or if free parameter } \{n \quad r\}$$

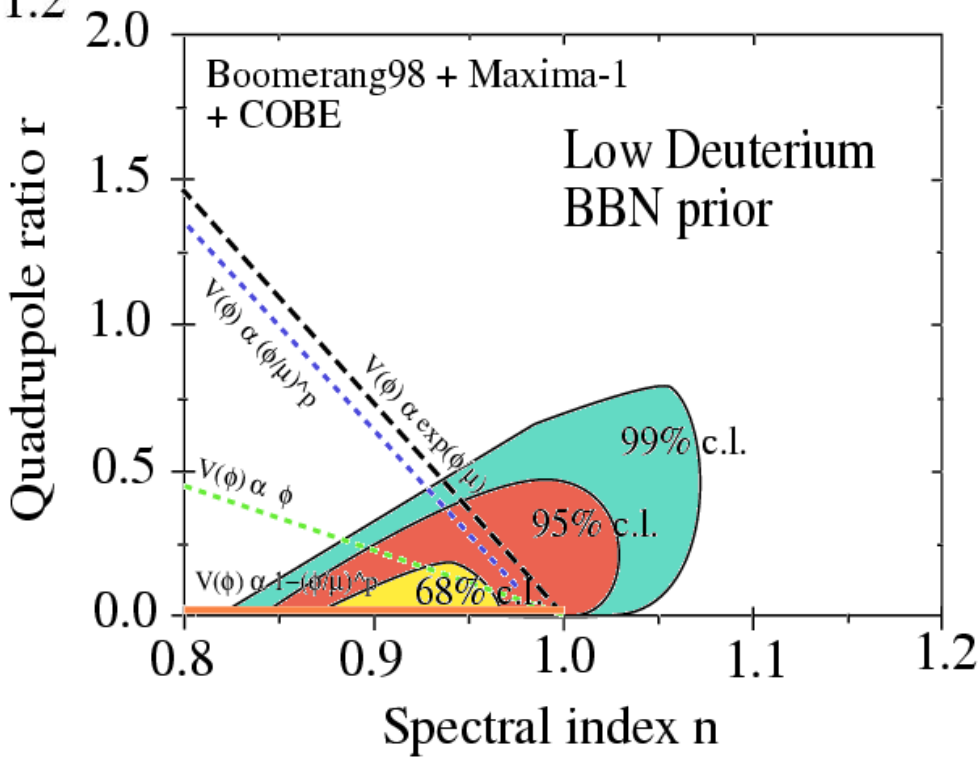
Dodelson, Kinney, Kolb astro-ph/9702156



$r = \left(\text{tensor}/\text{scalar}\right)_{l=2}$ $n = \text{scalar spectral index}$



Kinney, Melchiorri, Riotto
astro-ph/0007375



Harrison-Zel'dovich

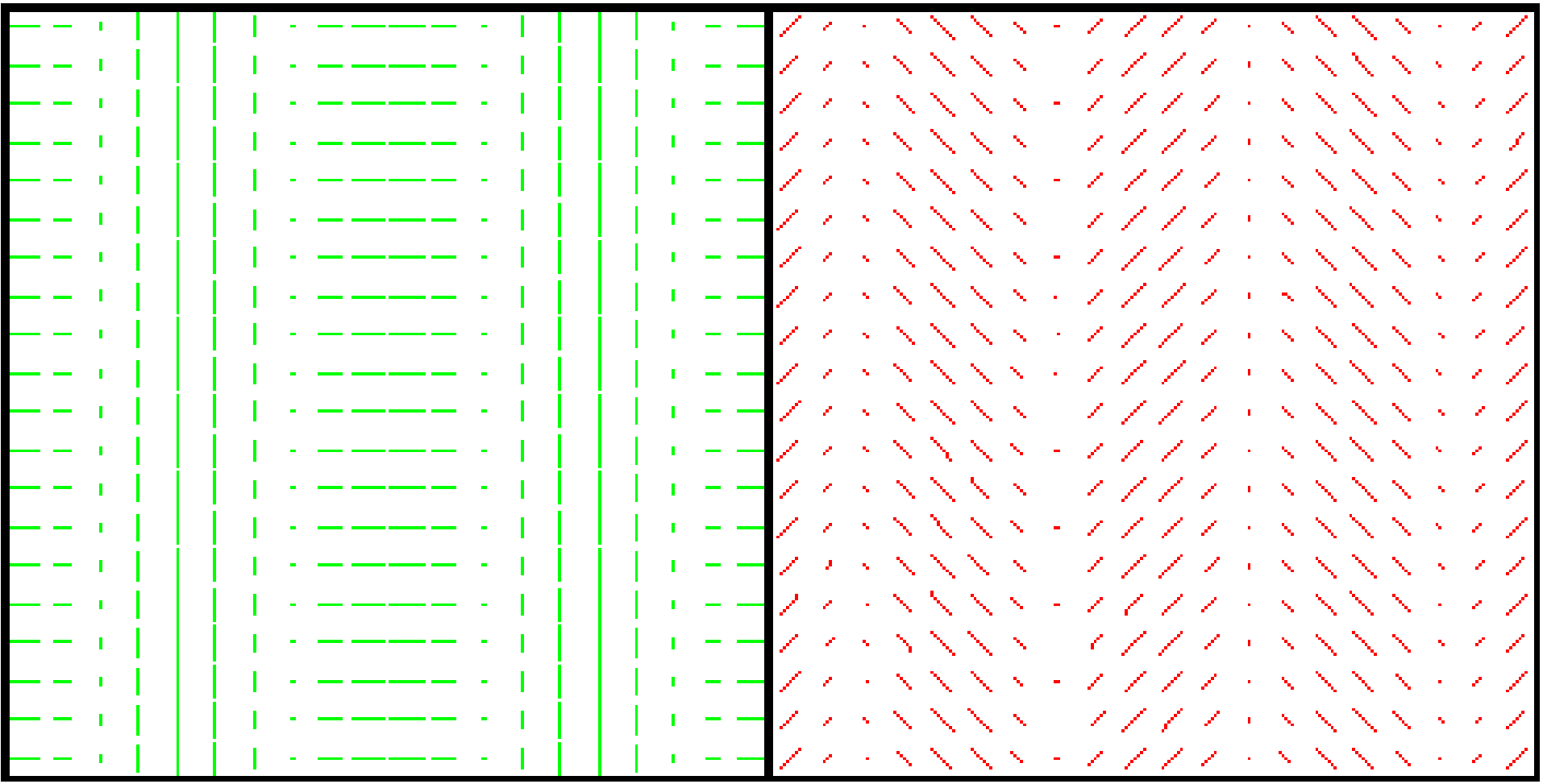
$n = 1.00000$
$r = 0.00000$

Fixed point of ignorance.

Polarization pattern

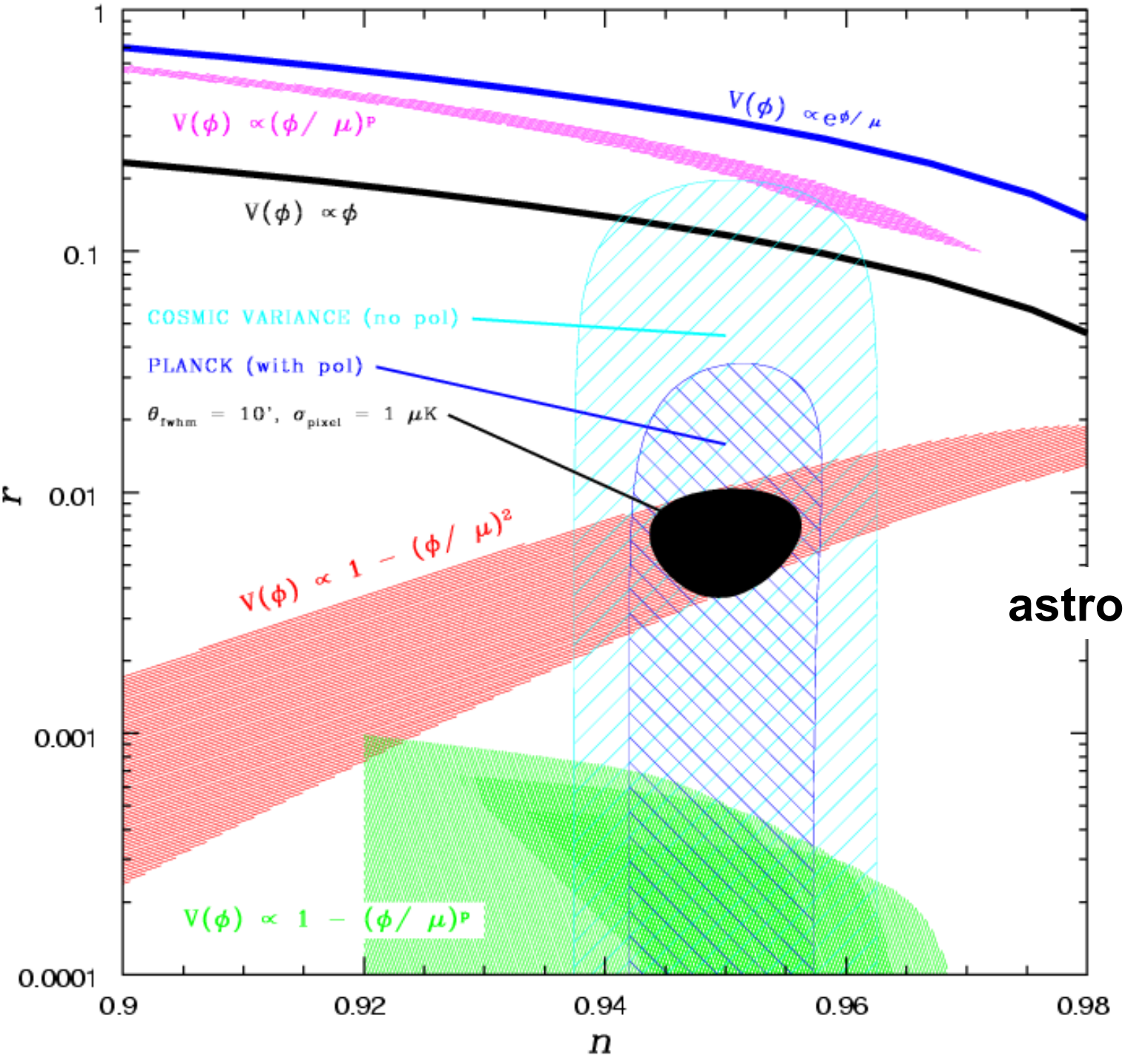
Stebbins, Kosowsky, Kamionkowski

Seljak & Zaldarriaga



E modes

B modes
(gravitational waves)



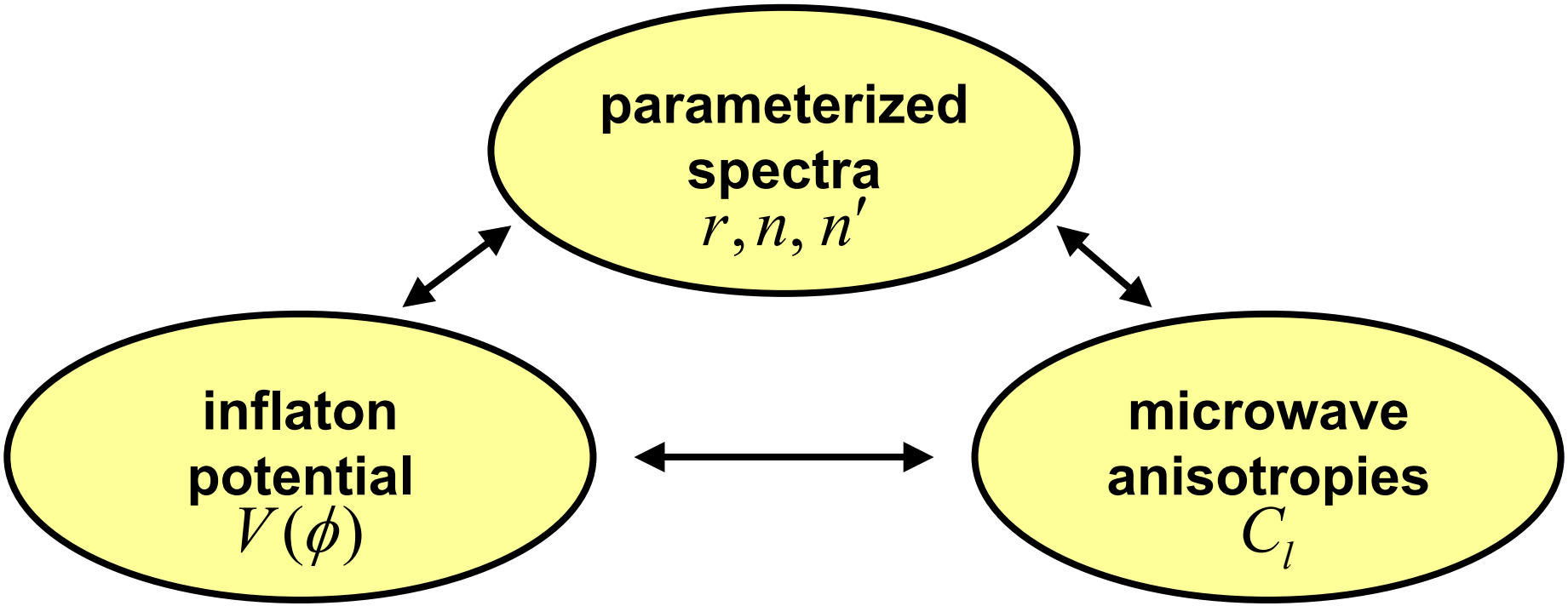
Kinney

astro-ph/9806259

Reconstruction

Bond, Abney, Copeland, Grivell, Kolb, Liddle, Lindsey, Turner, Sourdeep

Copeland, Kolb, Liddle, Lindsey Rev. Mod Phys. 97



Grivell & Liddle astro-ph/9906327

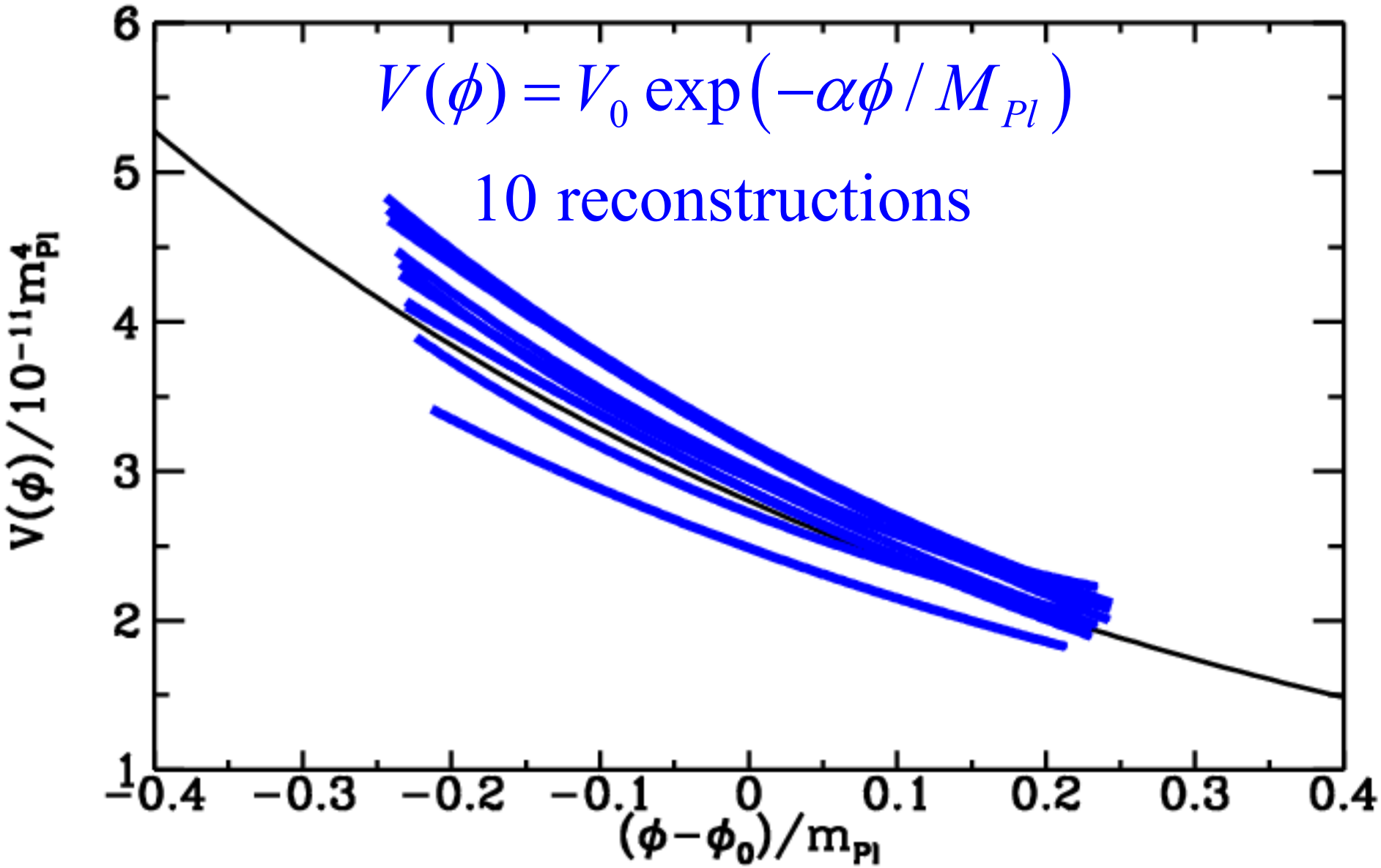
Reconstruction

$$\text{scalar} \sim \frac{V'(\phi)}{V(\phi)}$$

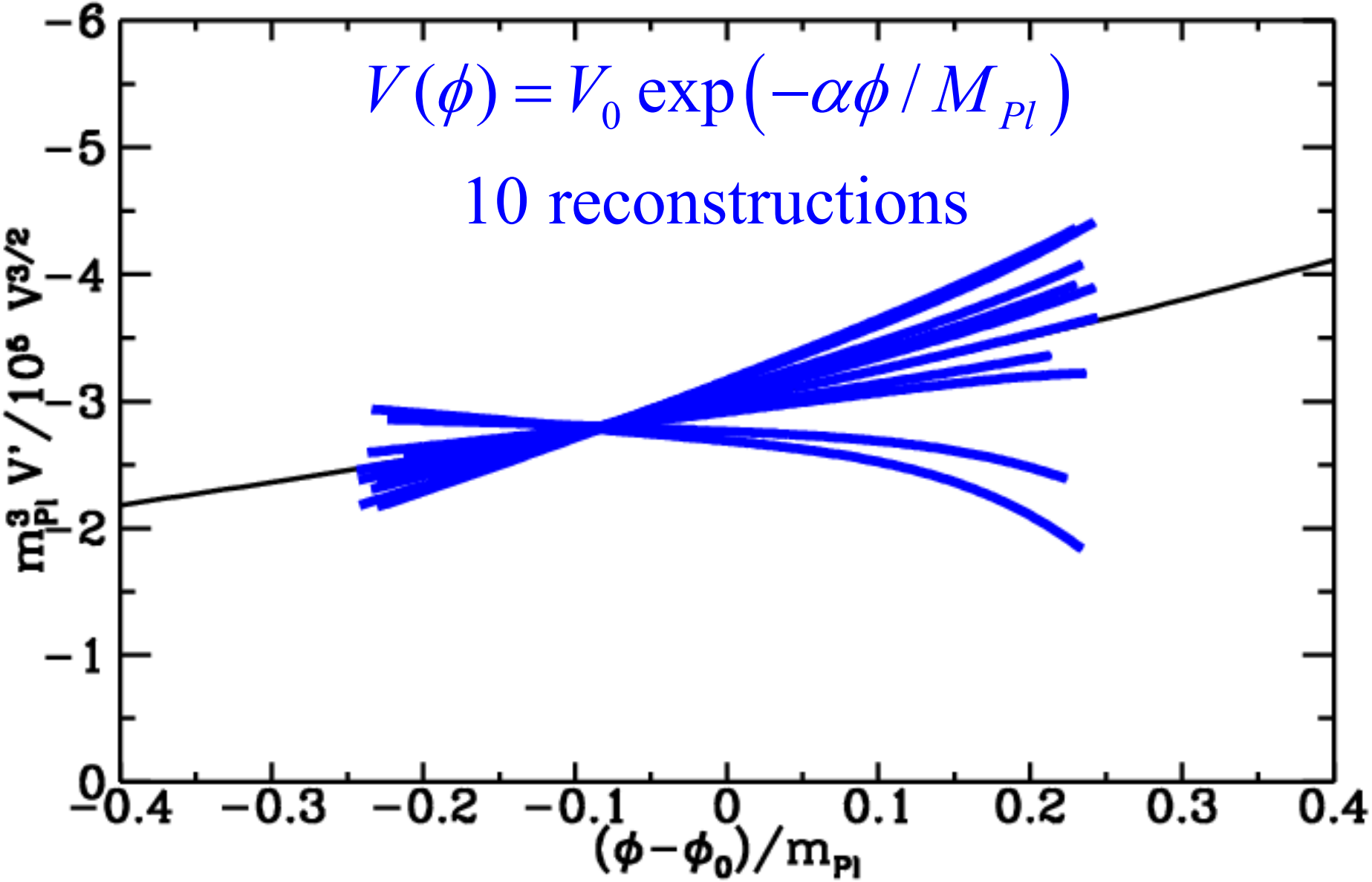
$$\text{tensor} \sim V(\phi)$$

1. **tensor spectral index in terms of scalar & tensor (consistency relation)**
2. **knowledge of the scale of *V* requires tensor**

Power-law spectrum



Power-law spectrum



Type I models predict*

1. a (nearly) exact *power-law*
2. spectrum of *gaussian*
3. *super-Hubble-radius*
4. *scalar* (density) and
5. *tensor* (gravitational-wave) perturbations
6. related by a *consistency relation*
7. in their *growing mode*
8. in a *spatially flat* universe.

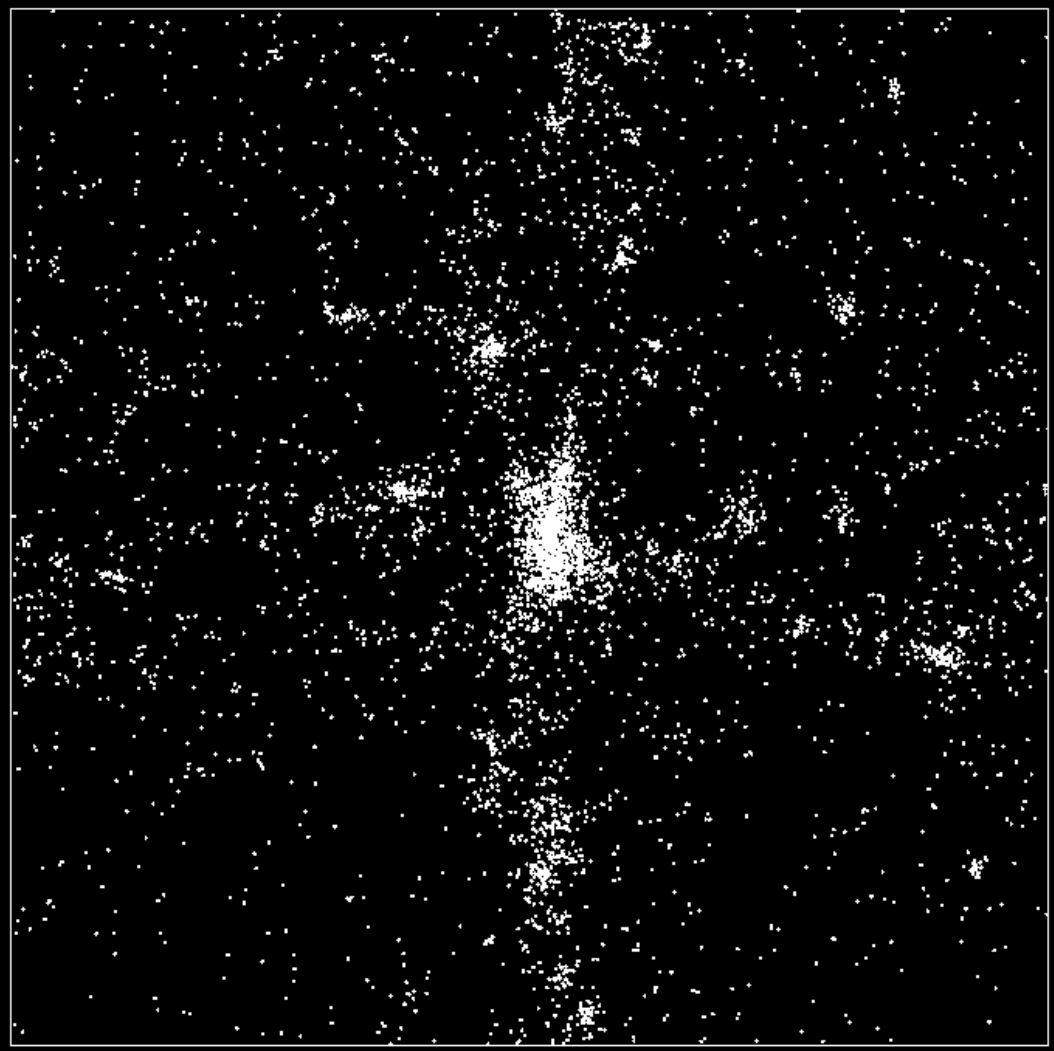
**at least the simplest ones*

Inflation conclusions

The alarming phenomenon of particle creation in the early universe can be studied by looking at the sky!

If you can look into the seeds of time
And say which grain will grow and which will not,
Speak then to me, who neither beg nor fear
Your favours nor your hate.

-MACBETH (Banquo)



Rocky III: Inflation

- Motivation for “acausal” perturbations
- Inflation in the cosmic symphony
- The alarming phenomenon of particle creation
- Inflation phenomenology

Cosmology and the origin of structure

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